Development of the 'Inner Assessment Model' of Long-Term Default Probability for Corporate Borrowers in the Trade Segment of the Economy in Accordance with IFRS 9

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Abstract

This work is the next step in the research project of various authors in modeling credit risk for Russian banks, taking into account the requirements of IFRS 9. This standard has been implemented all over the world since January 1, 2018 (including in the Russian banking market), and in accordance with the relevant standards it is necessary to clarify the existing models for assessing credit risk. IFRS 9 is based on the expected credit loss (ECL) approach. This new business model radically changes the approach to reserves under the rules of IFRS 9, including the impact of macroeconomic indicators on reserve value.

The purpose of this article is to create a model for assessing the probability of default for corporate borrowers in the trade 'industry' over the course of the whole life duration of assets, in accordance with the requirements of IFRS 9.

In this paper, the life-time probability of default of a financial instrument (referred to as life-time PD, or Lt PD) is based on a parametric model, and two distinct classes of distributions (the two-parameter Weibull distribution and the modified Weibull distribution) were studied. The results of model development are presented in this report.

The development of the model in this paper is based on real bank[1] data, so the results and methods used in this work can be applied by both commercial banks and regulatory authorities to model and implement the various requirements of IFRS 9. The practical value of this research also determines its scientific novelty, since this research is one of the first studies in the field of long-term probability of default using real data from Russian corporate clients of commercial banks.

Keywords: IFRS 9, expected credit losses, credit risk assessment stages, Weibull distribution
JEL classification: B40, G21, F65

[1] For confidentiality reasons, the authors do not disclose the name of the bank which portfolio data has been used, or the names of its clients.

Introduction

In the previous article under this research project, "Methods of Calculation of Expected Credit Losses under Requirements of IFRS 9" [1] and in other papers (e.g. [2]) we developed and presented the methodological concept of calculation of bank reserves for various types of assets, taking into consideration the specific character of requirements of IFRS 39. This article continues in the vein of the research papers written by the authors, studying the development of methods and approaches to credit risk modeling (i.e. assessment models of expected credit risk during the whole life-time of assets [3]). It is a project of development of the inner assessment model of the probability of default of corporate borrowers from the 'trade' segment of the economy for the whole life-time of the financial instrument, in accordance with requirements of IFRS 9.

This model has been developed on the basis of data from a real bank. Its results may be used from theoretical and practical points of view by commercial banks as well as by regulatory authorities when executing projects involving the implementation of IFRS 9 [4].

Defining Default

The conclusive evidence of a default is represented by the following circumstances:

- borrower's involuntary debt restructuring;
- payment delay for more than 90 days;
- assignment to the borrower of a rating of 10-default.

In general, when a customer approaches a bank for restructuring of a debt, the designated staff should first of all assign to the borrower a rating of 10-default, and then lift the default status, establishing that the customer is involved in the process of involuntary restructuring. Inasmuch as in practice there are cases of violation of this procedure, and that situations take place when the borrower is assigned a signal of involuntary restructuring without assigning the 10-default rating, it is generally accepted that the fact of involuntary restructuring is a conclusive evidence of default [5].

Description of the Approach to Development of the Model

At the time of development of the model evaluating the life-time probability of default of a financial instrument (Lt PD) of the trade segment of the economy (hereinafter the 'Trade segment') 36,213 observations were available (according to the key of entry "TIN + reporting date") concerning 1,507 borrowers since November 2011. When rating groups were made (see item 3.1), the number of observations in a rating group ranged from 716 observations concerning 22 borrowers to 7,557 observations concerning 159 borrowers (with the largest number of observations in "positive" rating groups and the smallest number of observations in "negative" rating groups). Besides this, there were a sufficient number of default observations for Lt PD modeling on the basis of empirical default rates (DR).

The approach to Lt PD modeling using a parametric model on the basis of internal data has the following advantages:

1) building of multiyear probability of default profiles on the basis of the observed default rates is an intuitive logical approach which does not require the articulation of additional suppositions as, for example, in the case of migration matrices;
2) the use of the approach helps (using the observed data) to extrapolate the results to any number of years (including nonintegrals) without overstating values of the last year's, which is characteristic of migration matrices. Therefore, this approach is preferable if default statistics are available for a sufficiently long period.

In view of these advantages, the approach to Lt PD modeling was chosen for the Trade segment on the basis of the parametric model, and under the scope of the project we studied two classes of distributions (the two-parameter Weibull distribution, and the modified Weibull distribution).

Stages of Modeling of Multiyear Probability of Default

Modeling of multiyear probability of default is based upon empiric (cumulative) default rates and existing probability of default (hereinafter: 'PD') (for 12 months) according to bank models.

The main stages of multiyear probability of default:

1) Obtaining cumulative default rates.
2) Building of multiyear cumulative through-the-cycle (TTC) PD profiles for each rating group on the basis of the Weibull distribution.
3) Converting to the master-scale, extrapolation and interpolation of the results of item 2.
4) Choice of macro parameters, and forms of dependence of default frequency on macro parameters.
5) Choice of scenarios for the modification of macro parameters.
6) Adjustment of TTC Lt PD obtained at the third stage, taking into consideration the macroforecast for defining marginal PD for the calculation of ECL.

Empiric Cumulative Default Rates

Annual default rate

The default rate (DR) was calculated for the first year, . This calculation was prepared with a breakdown into rating groups made for the purpose of modeling (see item 3.1). The equation is presented as follows:

$$DR(t) = \frac{\sum_{i=1}^{12} \text{New defaults}(t_{i-1};t_i)}{\text{Non}-\text{default customers}(t_0)}, \quad (1)$$

where t – an annual period;

t_0 – date of the beginning of the period;

$(t_{i-1}; t_i)$ – i-th month of period t;

DR(t) – one-year default rate in the year t.

The DR of the first year is calculated for the period of 01.11.2011–01.10.2016 (as of the same date in each month). In order to calculate the final one-year DR of the first year using the data from the reporting dates of 01.11.2011–01.10.2016, DR as at each date of the month (60 observations) was averaged. Averaging was conducted by calculating the arithmetic mean. The final DR is further used to calculate the cumulative default rates [5].

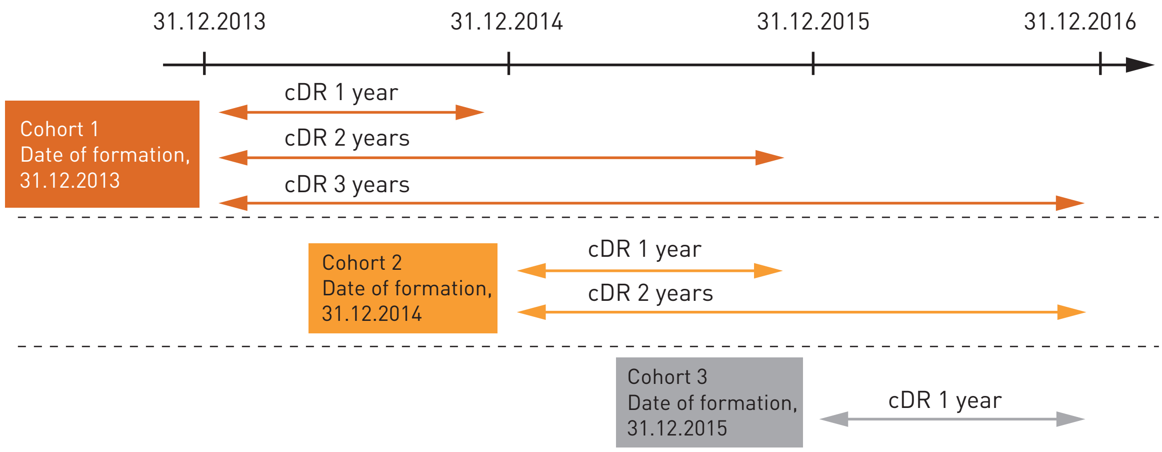
In a similar manner, we calculated the resulting DR for the second, third and fourth years. DR of the fifth year is not included in the further analysis due to a small number of observations (few customers stay in the portfolio till the fifth year, and the default rate of the fifth year tends toward zero).

Cumulative default rate

Cumulative default rates are calculated for each rating group for each year of the financial instrument life. Risk is assigned as at the date of the cohort formation.

Data is collected at the level of cohorts, where each cohort is formed as of the same date in each month. Examples of cohort formation are presented in Figure 1.

Figure 1. Cohort formation



Cumulative default rates are calculated on the basis of resulting DR as follows:

$$cDR_i = 1 - (1 - DR_1) \cdot (1 - DR_2) \cdot \ldots \cdot (1 - DR_i), \quad 2)$$

where cDR_i – cumulative default rate for i years;

DR_i – the final average one-year default rate in year i;

i – year.

Building of Multiyear TTC PD Profiles on the Basis of the Weibull Distribution [6]

Weibull distribution

The multiyear probability of default for the whole lifetime of a financial instrument is evaluated on the basis of the *Weibull distribution.*

The precondition for use of the Weibull distribution is the availability of data on historic default rates for the period of 3 to 8 years (in order to determine the distribution function parameters). This condition holds: data on four-year empirical cumulative default rates is available.

After obtaining empirical cumulative default rates, we calculate parameters k and λ of the two-parameter Weibull distribution function, or parameters α and β of the modified Weibull distribution.

Cumulative PDs are calculated on the basis of historic data on default.

In case of use of the Weibull distribution, the cumulative default rate cDR is described by the function of $F(\tau;\kappa,\lambda)$ – the two-parameter Weibull distribution function with parameters k and λ.

$$F(\tau;\kappa,\lambda) = \hat{e}DR(t,\lambda,\ ) = \begin{cases} 1 - e^{-\left(\frac{t}{\lambda}\right)^{\hat{e}}}, t \geq 0 \\ 0,\ t < 0 \end{cases}, \quad (3)$$

where k – the shape parameter;

λ – distribution function scale parameter;

t – period in years (integer, takes on the values of 1, 2, 3 etc.).

Parameters k and λ are defined for each rating group separately by linearisation of the dependency t and cDR_t:

$$cDR_t = 1 - e^{-\left(\frac{t}{\lambda}\right)^{\kappa}}$$

$$\ln(1 - cDR_t) = -\left(\frac{t}{\lambda}\right)^{\kappa}.$$

Substitution: assume $\kappa = b,\ \lambda = e^{-a/b}$

$$\ln(1-cDR_t) = -\left(\frac{t}{e^{-a/b}}\right)^b$$

$$b \cdot \ln t + a = \ln\left(-\ln\left(1-cDR_t\right)\right). \quad (4)$$

Parameters k and λ are assessed on the basis of a linear regression of the double logarithm of the survival function applying the least square method.

Parameters a and b of the linear regression are defined by the least square method:

$$\arg\min_{a,\,b}\left[\sum_{t=1}^{s}\left(y_t-\hat{y}_t\right)^2\right], \quad (5)$$

where $y_t = \ln\left(-\ln\left(1-cDR_t\right)\right)$;

$\hat{y}_t = a + b \cdot \ln\left(t\right)$;

s – the maximum period in years within which the default data is available;

$\sum_{t=1}^{s}\left(y_t-\hat{y}_t\right)^2$ – the sum of squared deviations.

On the basis of the obtained parameters, multiyear PD curves (TTC) are constructed individually for each rating group. In case where the approach is applied on the basis of the Weibull distribution, the following formula is representative:

$$cPD\left(t,\lambda_i,\kappa_i\right) = 1-e^{-\left(\frac{t}{\lambda}\right)^{\kappa}}, \quad (6)$$

where t – a period in years for which the probability of default is calculated (t = 1 is one year);

λ_i, k_i – Weibull distribution coefficients for rating group i.

The conditional probability of default PD(t) for year [t-1; t] is obtained from the cumulative probability of default using the following formula:

$$PD\left(t\right) = \frac{cPD\left(t\right)-cPD\left(t-1\right)}{1-cPD\left(t-1\right)}. \quad (7)$$

Modified Weibull distribution

In case of use of this modified Weibull distribution, the cumulative default rate cDR is described by the following function:

$$F\left(t,\alpha,\beta\right) = cDR\left(t,\alpha,\beta\right) = \begin{cases} \dfrac{1-e^{-\left(e^{-\left(\alpha\cdot t^{\beta}\right)}\right)}}{\left(1-e^{-1}\right)}, & t>0 \\ 0, & t\le 0 \end{cases} \quad (8)$$

where $\beta < 0$.

When using the modified Weibull distribution, the exponential approximation of the negative double natural logarithm of the survival function with the introduced coefficient and the natural logarithm of time is applied:

$$-\ln\left(-\ln\left(1-K \bullet cDR_t\right)\right) = \alpha \cdot e^{\beta\cdot\ln(t)}, \quad (9)$$

where $K = 1-e^{-1}$.

The exponential approximation takes into consideration the nonlinear nature of time distribution of the cumulative probability of default (due to the cyclical nature of economy, debt repayment etc.). The introduced coefficient K converts the expression into the function for which all four properties of the distribution function hold:

1. The function is right-continuous.
2. The function is not decreasing.
3. $\lim_{x\to-\infty} F\left(x\right) = 0$.
4. $\lim_{x\to+\infty} F\left(x\right) = 1$.

The linear dependence is obtained due to taking the logarithm in base e of both sides of equation:

$$cDR_t = \frac{1-e^{-\left(e^{-\left(\alpha\cdot t^{\beta}\right)}\right)}}{\left(1-e^{-1}\right)}$$

$$K = 1-e^{-1}$$

$$K \cdot cDR_t = 1-e^{-\left(e^{-\left(\alpha\cdot t^{\beta}\right)}\right)}$$

$$-\alpha \cdot e^{\ln t^{\beta}} = \ln\left(-\ln\left(1-K \cdot cDR_t\right)\right)$$

Substitution: assume $\alpha = e^A; \beta = B$

$$e^A \cdot e^{\ln t^B} = -\ln\left(-\ln\left(1-K \cdot cDR_t\right)\right)$$

$$A + B \cdot \ln t = \ln\left(-\ln\left(-\ln\left(1-K \cdot cDR_t\right)\right)\right). \quad (10)$$

Parameters A and B of the linear regression are defined by the least square method

$$\arg\min_{A,B}\left[\sum_{t=1}^{s}\left(y_t-\hat{y}_t\right)^2\right], \quad (11)$$

where

$y_t = \ln\left(-\ln\left(-\ln\left(1-K \cdot cDR_t\right)\right)\right)$;

$\hat{y}_t = A + B \cdot \ln\left(t\right)$;

s – the maximum period in years for which default data is available;

$\sum_{t=1}^{s}\left(y_t-\hat{y}_t\right)^2$ – sum of squared deviations.

On the basis of the obtained parameters, multiyear PD curves (TTC) are constructed individually for each rating group.

In case of using the approach on the basis of the modified Weibull distribution the following formula is applied:

$$cPD\left(t,\alpha_i,\beta_i\right) = \frac{1-e^{-\left(e^{-\left(\alpha_i\cdot t^{\beta_i}\right)}\right)}}{\left(1-e^{-1}\right)}, \quad (12)$$

where t – a period in years for which the probability of default is calculated (t = 1 is one year);

α_i, β_i – modified Weibull distribution coefficients for rating group i.

The conditional probability of default PD(t) for year [t-1; t] is obtained from the cumulative probability of default using formula (7).

Reducing to the Master-Scale, Extrapolation and Interpolation of the Results

After obtaining the cumulative probability of default profiles, the deviation of empiric default rates from the obtained curves is evaluated. Prolonged periods not covered by the available data are extrapolated, taking into consideration the following prerequisites:

1) curves cPD for various rating groups should not intersect in any time interval (see Figure 3);
2) the curves should not be too plane or too convex. This assessment is based on an expert opinion.

If nonmonotonicity is observed in the obtained cumulative probability of default profiles the probabilities of default should be converted into marginal probabilities of default for further elimination of intersections. (For nonmonotonicity in this instance, the following condition should be met: the probability of default of the worst rating exceeds the probability of default of the previous rating). Adjustment is made by assigning the maximum probability of default of previous ratings to the rating in which the monotonicity condition is violated.

Then the obtained monotonous marginal probabilities of default are transformed into cumulative ones.

The cumulative probabilities of default are converted into conditional probabilities of default for further translation of TTC PD for the first year into the bank master scale, and calculation of conditional PD for each rating inside the rating groups applying the logarithmic interpolation.

On the basis of the conditional PD obtained in the previous stage, we calculate the final marginal PD (without taking into consideration the forecasting information). Values of marginal PD are corrected in order to eliminate intersections.

The above transformations of the probability of default profiles are made using the following formulas:The cumulative PD is defined as follows:

$$CPD_t = \begin{cases} CPD_{t-1} + (1 - CPD_{t-1}) \cdot PD_t, t > 0 \\ 0, t = 0 \end{cases} . \quad (13)$$

The marginal PD is defined as follows:

$$MPD_t = PD_t \cdot (1 - CPD_{t-1}) = CPD_t - CPD_{t-1}. \quad (14)$$

Choosing the Approach

In order to choose a more accurate evaluation method of the probability of default for the life-time, the values of the coefficient of linear dependences determination (5) and (11) are compared. When the coefficient of linear dependence determination (11) exceeds the coefficient of linear dependence determination (5) the choice is in favour of the modified Weibull distribution. If the coefficient of linear dependence determination (11) is lower than the coefficient of linear dependence determination (5) the choice is in favour of the two-parameter Weibull distribution.

Construction of Lt PD PIT Model Taking into Consideration Forecasting Macroeconomic Information

One of the main requirements of the new Standard is ECL evaluation at a point in time (PIT) which implies use of historic data, current information, and forecasting information (macroeconomic factors). TTC PD is an average PD for the whole economic cycle, which evaluation is based on all available information on default rates for the whole available observation period. TTC PD is stable in time and has no correlation with the economic cycle [6].

PIT calibration is made on the basis of the Bayesian formula, where the PD of an agreement / customer / rating group is scaled according to the forecasting default rate and PD portfolio.

In order to transform the conditional one-year values of PD for each year of the financial instrument life-time, the Bayesian formula is applied as follows:

$$PD_i^{New} = \\ = \frac{(1 - CDT) \cdot DR_{New} \cdot PD_i}{CDT \cdot (1 - DR_{New}) \cdot (1 - PD_i) + (1 - CDT) \cdot DR_{New} \cdot PD_i}, \quad (15)$$

where PD_i^{New} is a new PD of the rating grade i which corresponds to a new default rate DR_{New}, taking into consideration the macroforecast for a corresponding year;

PD_i is a conditional PD of the rating grade i for a corresponding year (for the first year it corresponds to the bank master scale);

DR_{New} is the forecasting default rate for a corresponding year;

CDT is an average one-year default rate calculated according to the economic cycle.

Results of TTC Lt Pd Modeling before Taking into Consideration the Forecasting Macroeconomic Information

In order to evaluate ECL for the whole life-time of the financial instrument T, marginal probabilities of default are assessed for each life-time period of the financial instrument. Further the algorithm of obtaining these evaluations is described [7–8].

Results of Calculation of Empiric Cumulative Default Rates

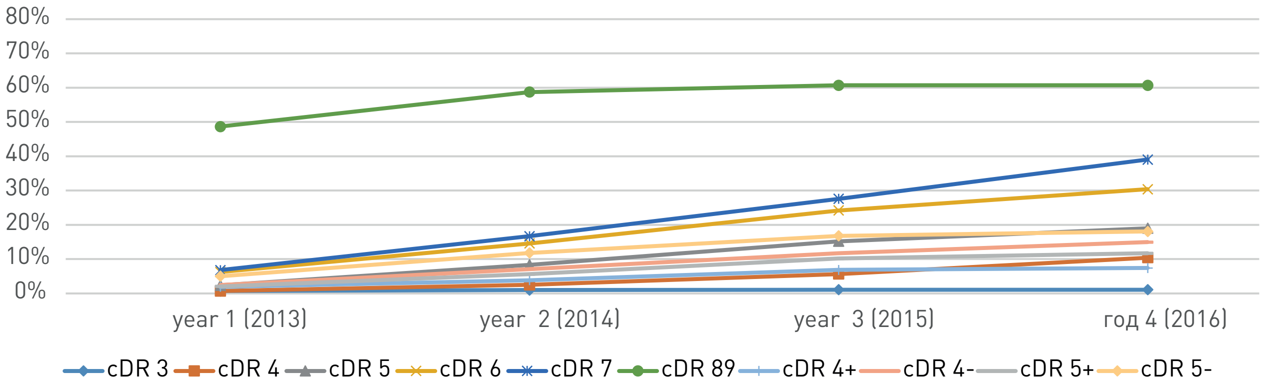
Cumulative default rates were calculated in accordance with formula (2):

- data from 01.11.2011 to 01.10.2016 (60 dates) was analysed; on the basis of the data Bank Default Register;
- DR in rating groups of the bank were calculated separately;
- for the Trade segment, the data was arranged into rating groups 3 (3+, 3, 3–), 4+, 4, 4–, 5+, 5, 5–, 6 (6+, 6, 6–), 7 (7+, 7, 7–), 89 (8+; 8; 8– and 9). The necessity to arrange the data into groups is explained by insufficient number of observations in individual rating grades (Table 1).

Table 1. Number of observations according to rating grades / groups.

Rating group	Rating grade	Number of observations in the rating grade	Number of observations in the rating group
3	1+	–	2,059
	1	–	
	1–	–	
	2+	1	
	2	–	
	2–	195	
	3+	149	
	3	485	
	3–	1,229	
4+	4+	2,450	2,450
4	4	2,791	2,791
4–	4–	3,784	3,784
5+	5+	4,120	4,120
5	5	4,237	4,237
5–	5–	4,268	4,268
6	6+	3,460	7,557
	6	2,461	
	6–	1,636	
7	7+	823	1,250
	7	295	
	7–	132	
89	8+	66	716
	8	519	
	8–	44	
	9	87	

Figure 2. Empiric cumulative default rates, Trade segment.



The initial ratings for the previous model (Corporate Customers, versions 2; 3.0; 3.1; 3.2) were counted (converted to the reference point of the Trade model – 4.4%). Therein we admit an assumption that the structure of the Corporate Customers models versions 2; 3.0; 3.1; 3.2 is similar to the structure of the Trade model, and such conversion calculation is admissible. The structure of the Corporate Customers model in version 1 differs significantly from the structure of the Trade model, and for this reason conversion calculation is impossible.

Figure 2 of Table 2 presents empiric cumulative default rates in the Trade segment.

Table 2. Empiric cumulative default rates for the Trade segment (%).

Rating group	Year 1	Year 2	Year 3	Year 4
3	0.68	0.96	1.07	1.07
4+	1.90	3.84	6.84	7.40
4	0.67	2.46	5.64	10.42
4–	2.41	7.08	11.71	14.95
5+	2.01	5.58	10.17	11.69
5	2.33	8.32	15.19	18.99
5–	4.99	11.79	16.78	18.06
6	6.23	14.57	24.19	30.37
7	6.77	16.67	27.57	39.04
89	48.64	58.73	60.70	60.70

Results of Construction of Multiyear Cumulative TTC PD Profiles on the Basis of the Weibull distribution and Choice of the Approach

In table 3–4, we present information on the obtained parameters of the function of the two-parameter Weibull distribution, and the modified Weibull distribution for the Trade segment in a bank calculated on the basis of the algorithm, described in section 2.

Table 3. Parameters of the two-parameter Weibull distribution function for the Trade segment.

Rating group	λ	κ
3	364 947.99	0.39
4+	39.91	1.07
4	12.64	1.97
4–	12.84	1.45
5+	15.54	1.42
5	8.95	1.71
5–	14.50	1.11
6	8.46	1.28
7	6.71	1.40
89	3.84	0.30

Table 4. Parameters of the modified Weibull distribution function for the Trade segment.

Rating group	α	β
3	5.44	–0.07
4+	4.42	–0.28
4	5.47	–0.47
4–	4.18	–0.44
5+	4.36	–0.40
5	4.21	–0.53
5–	3.44	–0.39
6	3.21	–0.51
7	3.13	–0.60
89	1.00	–0.29

Figure 3. Cumulative probabilities of default for the Trade segment.

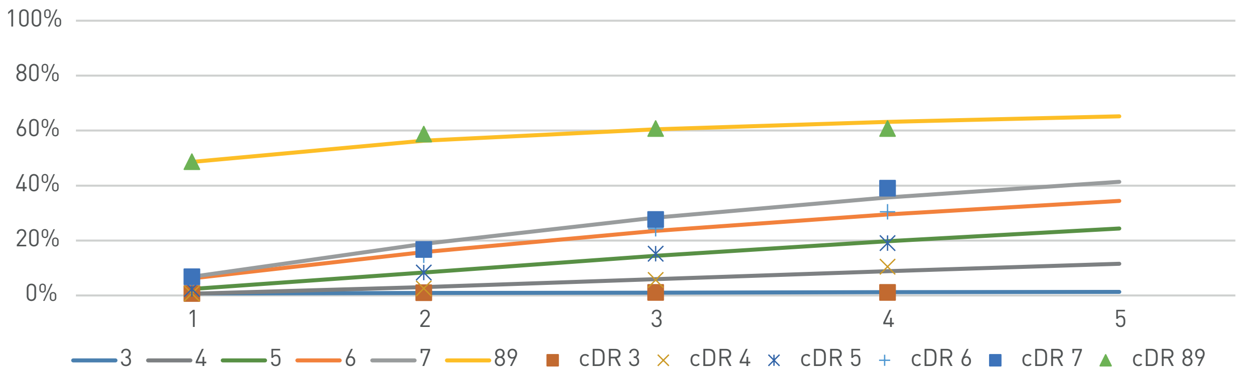


Figure 4. Comparison of the approach on the basis of the two-parameter Weibull distribution to the approach on the basis of the modified Weibull distribution using bank data (rating group "5-")

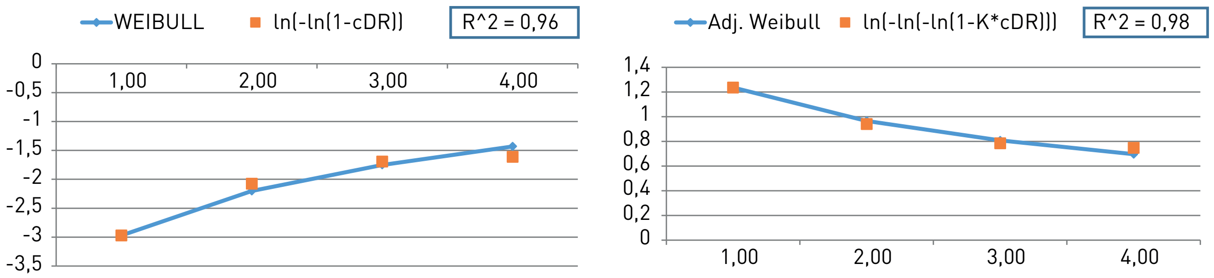


Figure 5. Marginal probabilities of default for the Trade segment, rating group "5-".On the basis of the obtained parameters we built multiyear PD curves (TTC) separately for each rating group (Figure 3).

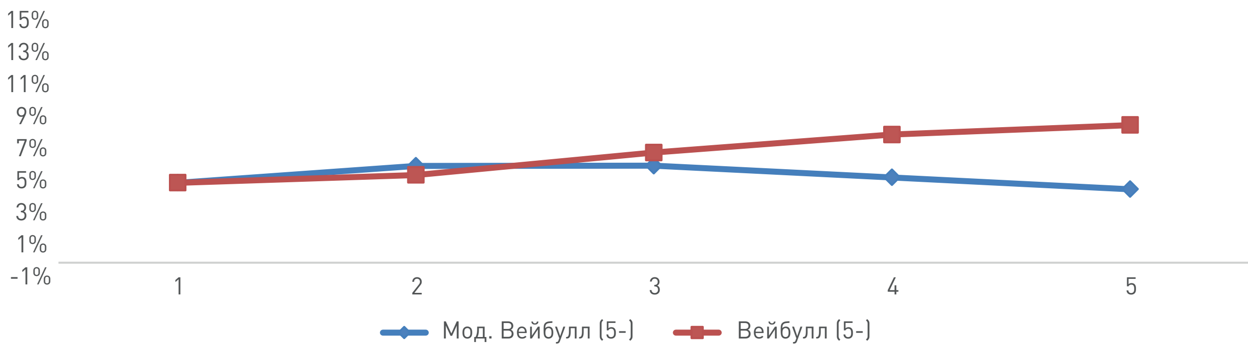


Figure 4 represents by a diagram comparison of two methods using rating group "5-" as an example (in the Trade segment it has a rather high concentration of borrowers).

As long as the determination coefficient for the two-parameter Weibull distribution is a little lower than the coefficient for the modified Weibull distribution (0.96<0.98) a decision was taken to choose the modified two-parameter Weibull distribution.

For rating groups 3, 4–, 5+, 5, 5– where the borrower concentration amounts to 57.7% the determination coefficient for the two-parameter Weibull distribution is lower than the coefficient for the modified Weibull distribution. So, on the basis of a comparison of the results of the two methods for other rating groups a decision was taken in favour of the modified two-parameter Weibull distribution.

Reducing to the Master-Scale, Extrapolation and Interpolation of the Results

It is evident from Table 5 that in the theoretical cumulative probabilities of default calculated according to formula (12) using the obtained parameters α and β non-monotonicity is observed (for example, probabilities of default for rating "4+" exceed the probabilities of default of rating "4", probabilities of default for rating "4-" exceed the probability of default of rating "5+").

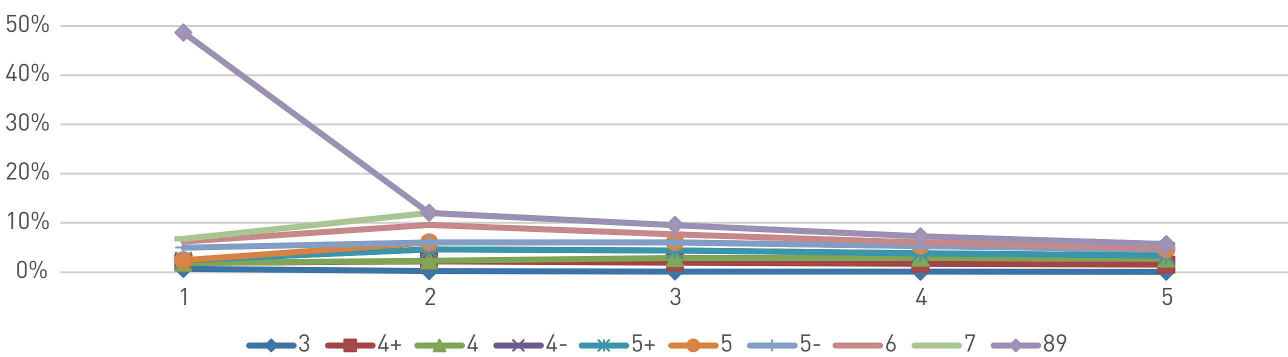
Table 5. Cumulative default rates for the Trade segment evaluated on the basis of the modified Weibull function (%).

Rating group	Year 1	Year 2	Year 3	Year 4	Year 5
3	0.68	0.90	1.04	1.16	1.26
4+	1.90	4.13	6.08	7.78	9.29
4	0.67	3.00	5.89	8.80	11.57
4–	2.41	7.05	11.44	15.31	18.70
5+	2.01	5.72	9.28	12.47	15.29
5	2.33	8.37	14.43	19.76	24.35
5–	4.99	11.04	15.94	19.93	23.27
6	6.23	15.81	23.49	29.54	34.40
7	6.77	18.81	28.36	35.65	41.34
89	48.64	56.30	60.44	63.20	65.23

Table 6. Marginal probabilities of default for the Trade segment after elimination of nonmonotonicity (%).

Rating group	Year 1	Year 2	Year 3	Year 4	Year 5
3	0.68	0.21	0.15	0.12	0.10
4+	1.90	2.23	1.95	1.70	1.51
4	1.90	2.34	2.88	2.91	2.77
4-	2.41	4.63	4.40	3.87	3.38
5+	2.41	4.63	4.40	3.87	3.38
5	2.41	6.04	6.06	5.33	4.59
5–	4.99	6.05	6.06	5.33	4.59
6	6.23	9.58	7.69	6.04	4.86
7	6.77	12.04	9.56	7.29	5.69
89	48.64	12.04	9.56	7.29	5.69

Figure 6. Marginal Probabilities of default for the Trade segment.



The following transformations have been consistently applied to the obtained cumulative probabilities of default for the Trade segment.

Step 1. Cumulative probabilities of default were converted into marginal probabilities of default (see formula (14)) for further elimination of intersections. Adjustment was made by assigning the probability of default of the previous rating to the rating in which the monotoneness condition is violated.

Marginal TTC profiles of multiyear probabilities of default were adjusted to eliminate intersections (for rating groups 4, 5+, 5– and 89 mPD they were fixed at the level of maximum mPD for rating groups 4+, 4–, 5 and 7, respectively).

Marginal probabilities of default after the adjustments are represented in Table 6.

Step 2. Marginal probabilities of default were transformed into cumulative ones (in accordance with dependency (13)).

Cumulative probabilities of default after adjustment are represented in Table 7.

Table 7. Cumulative probabilities of default for the Trade segment after elimination of nonmonotonicity (%).

Rating groups	Year 1	Year 2	Year 3	Year 4	Year 5
3	0.68	0.90	1.04	1.16	1.26
4+	1.90	4.13	6.08	7.78	9.29
4	1.90	4.24	7.12	10.03	12.80
4–	2.41	7.05	11.44	15.31	18.70
5+	2.41	7.05	11.44	15.31	18.70
5	2.41	8.45	14.51	19.84	24.43
5–	4.99	11.04	17.10	22.43	27.02
6	6.23	15.81	23.49	29.54	34.40
7	6.77	18.81	28.36	35.65	41.34
89	48.64	60.68	70.23	77.52	83.21

Table 8. Conditional multiyear probabilities of default for the Trade segment (%).

Rating group	Empiric PD TTC	Year 1 (PD according to the bank master-scale)	Year 2	Year 3	Year 4	Year 5
3	0.68	0.58	0.82	0.73	0.65	0.59
4+	1.90	0.96	2.28	2.03	1.81	1.64
4	1.90	1.23	2.38	3.01	3.13	3.08
4–	2.41	1.58	4.75	4.73	4.37	3.99
5+	2.41	2.03	4.75	4.73	4.37	3.99
5	2.41	2.61	6.19	6.62	6.23	5.73
5–	4.99	3.36	6.37	6.81	6.43	5.92
6	6.23	5.54	10.21	9.13	7.90	6.90
7	6.77	11.74	12.91	11.77	10.17	8.84
89 (8–)	48.64	31.97	42.20	24.30	24ю48	25.30

Step 3. Cumulative probabilities of default were transformed into conditional probabilities of default (in accordance with dependence (7)) for further conversion of TTC PD for the first year to the bank master-scale, and calculation of conditional PD for each rating inside rating groups by means of logarithmic interpolation (stage 4).

Moreover, as long as the conditional probabilities of default for rating group "3" in the second and subsequent years are significantly lower than in the first year (judged on the basis of the empiric selection data) they were adjusted for the correction factor of change of the conditional probability of default from year to year in rating group "4+".

The bank uses a fixed scale of the bank inner rating mapping with PD_{TTC}. The obtained probabilities of default for the first year were replaced in accordance with the bank master-scale (Table 8).

Step 4. Conditional PD for each rating inside rating groups 3, 6, 7, 89 were calculated by means of logarithmic interpolation.

Example of calculation of logarithmic interpolation for rating "3-" (year 2):

$$PD_{3-} = PD_3 \bullet \left(\frac{PD_{4+}}{PD_3}\right)^{1/2}, \qquad (16)$$

where (1/2) – ratio of the distance (number of rating grades) between the target value PD3 and the known value PD3 (equals one) to the distance between two known values PD3 and PD4+ (equals two).

Conditional PD for ratings 1+, 1, 1–, 2+, 2 and 2– are fixed at the level of the bank master-scale (Table 9).

Table 9. Conditional multiyear probabilities of default for the Trade segment after applying logarithmic interpolation (%).

Risk category	Year 1	Year 2	Year 3	Year 4	Year 5
1+	0.01	0.01	0.01	0.01	0.01
1	0.02	0.02	0.02	0.02	0.02
1–	0.04	0.04	0.04	0.04	0.04
2+	0.08	0.08	0.08	0.08	0.08
2	0.16	0.16	0.16	0.16	0.16
2–	0.32	0.32	0.32	0.32	0.32
3+	0.45	0.49	0.44	0.39	0.35
3	0.58	0.82	0.73	0.65	0.59
3–	0.75	1.36	1.22	1.09	0.98
4+	0.96	2.28	2.03	1.81	1.64
4	1.23	2.38	3.01	3.13	3.08
4–	1.58	4.75	4.73	4.37	3.99
5+	2.03	4.75	4.73	4.37	3.99
5	2.61	6.19	6.62	6.23	5.73
5–	3.36	6.37	6.81	6.43	5.92
6+	4.31	8.06	7.88	7.13	6.39
6	5.54	10.21	9.13	7.90	6.90
6–	7.12	11.04	9.94	8.60	7.49
7+	9.14	11.94	10.81	9.35	8.14
7	11.74	12.91	11.77	10.17	8.84
7–	15.08	17.36	14.11	12.67	11.50
8+	19.37	23.34	16.91	15.78	14.95
8	24.89	31.39	20.27	19.66	19.45
8–	31.97	42.20	24.30	24.48	25.30
9	41.06	56.73	29.13	30.49	32.91

Step 5. On the basis of conditional PD obtained at step 4, marginal PD values were calculated (see formula (14)).

For ratings 1+, 1, 1–, 2+, 2 and 2– mPD were fixed at the level of the bank master-scale.

For ratings 5+, 8, 8– and 9 starting from the third year the values of marginal PD were adjusted to eliminate intersections.

The final mPD values (without taking into consideration forecasting information) are presented in Table 10.

Table 10. Marginal TTC profiles of multiyear probabilities of default for the Trade segment (%).

Rating group	Year 1	Year 2	Year 3	Year 4	Year 5
1+	0.01	0.01	0.01	0.01	0.01
1	0.02	0.02	0.02	0.02	0.02
1–	0.04	0.04	0.04	0.04	0.04
2+	0.08	0.08	0.08	0.08	0.08
2	0.16	0.16	0.16	0.16	0.16
2–	0.32	0.32	0.32	0.32	0.32
3+	0.45	0.49	0.43	0.39	0.35
3	0.58	0.81	0.72	0.64	0.57
3–	0.75	1.35	1.19	1.05	0.94
4+	0.96	2.25	1.97	1.72	1.52

Rating group	Year 1	Year 2	Year 3	Year 4	Year 5
4	1.23	2.35	2.90	2.93	2.79
4–	1.58	4.67	4.43	3.90	3.41
5+	2.03	4.67	4.43	3.90	3.41
5	2.61	6.03	6.04	5.32	4.58
5–	3.36	6.15	6.16	5.42	4.67
6+	4.31	7.72	6.94	5.78	4.81
6	5.54	9.65	7.74	6.09	4.89
6–	7.12	10.26	8.21	6.40	5.09
7+	9.14	10.85	8.65	6.67	5.26
7	11.74	11.40	9.05	6.90	5.38
7–	15.08	14.74	9.90	7.64	6.05
8+	19.37	18.82	10.45	8.11	6.47
8	24.89	23.57	10.45	8.11	6.47
8–	31.97	28.71	10.45	8.11	6.47
9	41.06	33.44	10.45	8.11	6.47

Construction of pit lt pd Model Taking into Consideration Forecasting Macroeconomic Information

Choice of Macro Parameters and Forms of Dependence of Default Frequency on Macro Parameters for Further Analysis [10–12]

Choice of explicative variables

In order to analyse the macroeconomic information the integrated data as regards the Trade segment as well as Manufacturing and Service segments of the economy were used for the purpose of evening-out the high volatility of DR in one of the segments - Manufacturing and Services – which is not related to relevant macroeconomic factors.

Macroeconomic factors forecasted by the bank were used as independent variables. Analysis was carried out as at the dates of 01.01.2012 to 01.10.2016 (58 monthly dates).

"Long List" of explicative variables

In order to choose the parameter which demonstrates the maximum dependence as regards the default frequency, the following converted indicators were analysed:

- annual consumer price index for goods and services for the following 12 months from the reporting date, % (source: <www.gks.ru>) – CPI_1;
- increment of the annual consumer price index for goods and services for the following 12 months to the previous 12 months as of a date, % (source: <www.gks.ru>) – CPI_2;
- annual index of change of the interest rate of the Central Bank of the Russian Federation, as a % of the following 12 months from the reporting date (source: <www.cbr.ru>) – cb_rate;
- real disposable household income, as a % of the corresponding period of the previous year (source: <www.gks.ru>) – rdi_1;
- average annual index of the real disposable household income, as % for the last 12 months (source: <www.gks.ru>) – rdi_2;
- index of change of average annual prices for Brent oil, for the following 12 months from the reporting date, % (source: <finam.ru>) – oil_aver;
- increment of the average annual exchange rate of the US dollar, for the following 12 months from the reporting date, % (source: <finam.ru>) – dollar_aver;
- increment of the exchange rate of the US dollar as of the end of the period, for the following 12 months from the reporting date, % (source: <finam.ru>) – dollar_ep;
- increment of the average annual exchange rate of the Euro, for the following 12 months from the reporting date, % (source: <finam.ru>) – euro_aver;
- increment of the exchange rate of the Euro as of the end of the period, for the following 12 months from the reporting date, % (source: <finam.ru>) – euro_ep;
- average annual index – GDP deflator, for the following 12 months from the reporting date, % (source: <www.gks.ru>) – gdp_deflator;
- annual increment of GDP in roubles in constant prices, for the following 12 months from the reporting date, % (source: <www.gks.ru>) – gdp_1;
- annual increment of GDP in US dollars in constant prices, for the following 12 months from the reporting date, % (source: <www.gks.ru>) – gdp_2.

Although the GDP deflator index is not forecasted in the bank, this indicator was used for analysis because it illustrates very well the pattern of economic development of the Russian Federation, and the publicly-available forecast is based on it (Department of Macroeconomic Analysis and Forecasting of the Ministry of Economic Development of the Russian Federation[1]).

All indicators, except for rdi_1, rdi_2, are considered "for the following 12 months from the reporting date", and this corresponds to the period for which the default rates DR are calculated. Apart from that, for the purposes of

[1] URL: http://economy.gov.ru/minec/activity/sections/macro/prognoz/

taking into account macroeconomic information, the forecasting future values of the chosen explanatory factor are used. For the indicator of the "real disposable household income, as a % of the corresponding period of the previous year", the logic of "for the last 12 months" was preserved in order to obtain economically interpretable negative correlation ratios of indicators rdi_1, rdi_2 with the default rates DR.

In order to take into consideration the deferred influence of the factors the indicators with the lags of 1–18 months were considered.

Selection Procedure

Correlation Analysis

Linear interrelation ratios between variables (Pearson correlation) were calculated. The obtained results were analysed in order to select macroeconomic factors with the biggest linear interrelation value with the default rate.

In Table 11, the variables with the biggest correlation ratio with a dependent variable are presented.

Correlation was calculated as follows:

- by pairs between macroeconomic factors;
- between the dependent variable and macroeconomic factors;
- between the dependent variable and lag values of macroeconomic factors.

When selecting factors for further analysis we used the following criteria:

- coherence of the pair correlation sign with the economic sense of variables interrelation;
- pair correlation ratio of independent variables does not exceed 60% (in modulus);
- factor / factor lag correlation ratio with a dependent variable takes on the maximum value.

Table 11. Correlation ratios between selected variables from the list of factors and the default rate (in the segments of Trade, Manufacturing, and Services).

Variable	Variable description	Significant lag in months	Correlation with c DR, %
dollar_aver_lag4	Increment of the average annual exchange rate of the US dollar, for the following 12 months from the reporting date, %	4	93.3
CPI_1_lag6	Annual consumer price index for goods and services for the following 12 months from the reporting date, %	6	92.3
oil_aver _lag1	Index of change of average annual prices for Brent oil, for the following 12 months from the reporting date, %	1	–88.9
dollar_ep_lag8	Increment of the exchange rate of US dollar as of the end of the period, for the following 12 months from the reporting date, %	8	87.1
gdp_1	Annual increment of GDP in roubles in constant prices, for the following 12 months from the reporting date, %	0	–87.1

The results of these calculations show a strong linear interrelation between the default rates in the Trade segment and the Manufacturing and Services segment and five selected macroeconomic variables in the period of 01.01.2012 to 01.10.2016.

Graphical Analysis

The graphical analysis does not confirm a powerful influence of the consumer price index (CPI_1_lag6) and increment of US dollar exchange rate (dollar_ep_lag8) on the default rates (there are concentrations of points in two areas. Inside these areas there is no correlation relationship between indicators, or it has the sign opposite to the total correlation ratio). Thus, on the basis of the results of the graphical analysis, the following factors have been selected: 1) GDP annual increment; 2) increment of the average annual US dollar exchange rate; 3) index of change of average annual prices for Brent oil. In graphical analysis, these indicators show a high correlation with the default rates and a well-defined linear trend. These macroeconomic indicators also have a good interpretability with regard to influence on the default rates.

See below dependence diagrams according to DR type (macro factor) (figures 7–11).

Figure 7. Increment of the average annual exchange rate of the US dollar, for the following 12 months from the reporting date, %

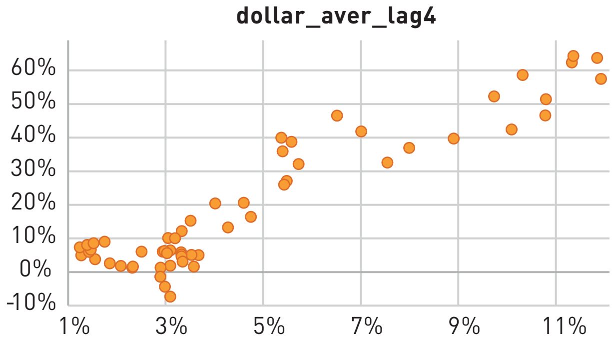


Figure 8. Annual consumer price index for goods and services for the following 12 months from the reporting date, %

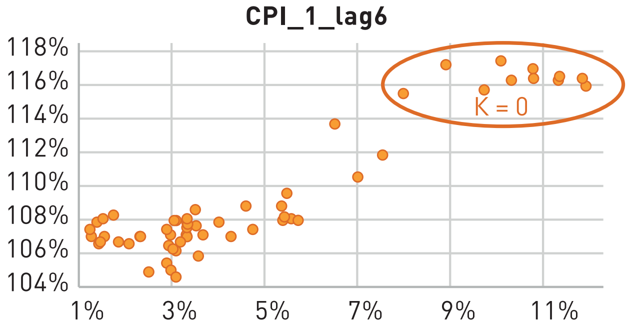


Figure 9. Index of change of average annual prices for Brent oil, for the following 12 months from the reporting date, %

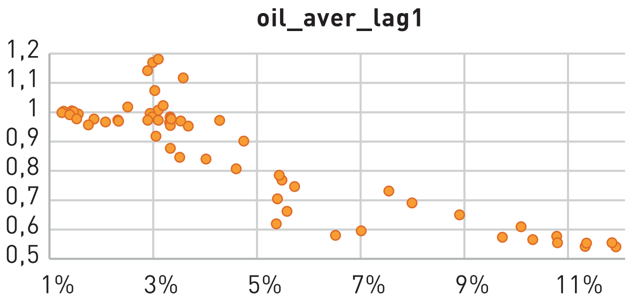


Figure 10. Increment of the exchange rate of the US dollar as of the end of the period, for the following 12 months from the reporting date, %

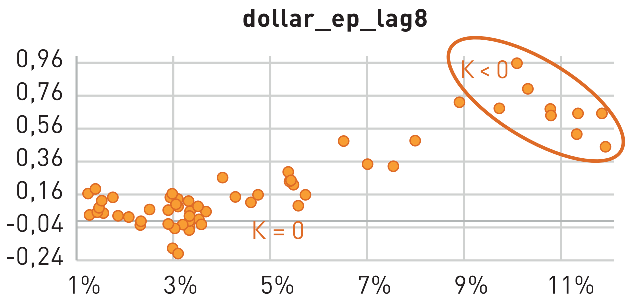
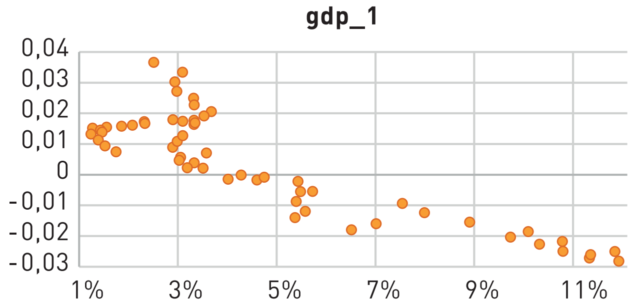


Figure 11. Annual increment of GDP in roubles in constant prices, for the following 12 months from the reporting date, %



"Short List" of explicative variables:

- annual increment of GDP in roubles in constant prices, for the following 12 months from the reporting date, % (source: <www.gks.ru>) – gdp_1;
- increment of the average annual exchange rate of US dollar, for the following 12 months from the reporting date, % (source: <finam.ru>) – dollar_aver;
- index of change of average annual prices for Brent oil, for the following 12 months from the reporting date, % (source: <finam.ru>) – oil_aver.

Econometric Analysis

Stage 1. Construction of One-Factor Models

Due to existence of high correlation factors with an independent variable, the following one-factor models were constructed: linear, log-linear and Vasicek models. The following models showed the greatest forecast power:

1) Vasicek model with the variable dollar_aver_lag4 (R^2 = 88.8%);
2) Vasicek model with the variable gdp_1 (R^2 = 88.3%);
3) log linear regression model with the variable dollar_aver_lag4 (R^2 = 87.4%).

The Vasicek model with the variable dollar_aver_lag4h showed the greatest forecast power, a similar model with the variable gdp_1 showed a slightly smaller forecast power (R^2 is less by 0.5%). After discussion a study group defined that the model on the basis of the factor of increment of the average US dollar exchange rate is less interpretable than the model with the GDP increment rate variable. Taking into consideration the results of the graphical analysis, the Vasicek model with the GDP increment rate variable was chosen as the best model on the basis of an analysis of all one-factor models.

Stage 2. Construction of Two-Factor Models

In order to verify whether it is reasonable to add the second factor to the model in order to strengthen its forecasting properties, the strongest two-factor models were built, taking into consideration the following limitations:

- correlation between the factor and default rate (in modulus) not less than 40% (substantiation of existence of influence on the default rate);
- correlation between factors (in modulus) not exceeding 60% (absence of multicollinearity);

- deviation of the correlation ratio between the factor and DR from the maximum correlation value of this factor with the optimal lag value not exceeding 0.1 (substantiation of choice of the lag length)[2];
- each factor in the model is statistically significant: p-value not exceeding 0.05;
- we excluded certain models based on the factors with poor properties on the basis of the results of the graphical analysis: dollar_ep, and CPI_1;
- we excluded the models in which the sign of the regression coefficient in front of the variable does not correspond to the economic sense of influence of the variable on the default rate: rdi2_lag18, rdi2_lag17 etc.

The model of log linear regression ($R^2adj = 92.7\%$) with the variables: dollar_aver_lag2 (increment of the average annual US dollar exchange rate) and rdi_2_lag3 (average annual index of real disposable household income) showed the greatest forecasting power. The predictive power of this model is somewhat greater than that of the one-factor model with the variable gdp_1 ($R^2 = 88.3\%$) but adding supplementary factors to the model did not result in a significant increase of its predictive power, and therefore is unreasonable.

Stage 3. Back Testing of the Model

We verified the results of one-factor models with the variable of "GDP growth rate" in the test selection (back testing) applying the following approach: distinguishing of an individual training selection for development of the model and a testing selection to verify its quality[3]. The final model should meet the following criteria:

- the model should have the highest determination coefficient;
- the relative reduction of R^2 in the testing selection should not exceed 5%.

The results of comparison of the approaches to analysis are presented in Table 12.

Table 12. Results of choice of the approach to building of the final model (%).

Model	R^2 in the selection:	
	Training selection	Testing selection
Vasicek model	89.3	86.7
Log linear regression	86.1	83.6
Linear regression	77.1	73.7

On the basis of the results of the analysis, we see that the Vasicek model represents most adequately the influence of the annual GDP increment rate on the default rate in the segments of Trade, Manufacturing, and Services. This model is highly efficient in the training selection, $R^2 = 89.3\%$. In the testing selection the efficiency of the model is slightly lower, $R^2 = 86,7\%$. Reduction in effectiveness of a selection within 5% is admissible.

Stage 4. Model Parameters Evaluation

The constructed Vasicek model is written as:

$$DR = N\left(\frac{N^{-1}\left(DR_{avg}\right) - \sqrt{\rho}\, Z}{\sqrt{1-\rho}}\right), \qquad (17)$$

where $N()$ – standard normal distribution;

$N^{-1}()$ – inverse normal distribution;

DR – default rate;

DR_{avg} – average level of DR across the selection;

Z – standardised value of macroeconomic factor calculated by the following formula:

$$Z = \frac{X - \overline{X}}{\sigma},$$

where X – macro factor value as of the reporting date;

$\overline{X}$ – macro factor average value;

σ – standard deviation of macro factor;

ρ – model parameter characteristic of the level of the nonlinear dependence between the macro factor value and DR.

The value of parameter ρ was evaluated on the basis of the condition of maximisation of the total determination coefficient of the model (using the 'Solver' add-in in MS Excel). The stability of the predictive power of the model on the basis of indicator R^2 on an annual basis was thereby controlled. This approach helped to maximise the total R^2 up to88.3% and achieve a high predictive power of the model since 2013 (since 2013 $R^2 \geq 87.7\%$). The insufficient predictive power of the model using the data for 2012 may be explained by a high DR volatility in the segment of Manufacturing and Services in this year, not related to macroeconomic factors. Table 13 presents information on th estability of the R^2 indicator arranged by the years.

[2] This is about deviation in the correlation ratio value when a lag variable, for which correlation with DR is less than for the variable with the optimal lag length, is used in the model. For example, assume that the maximum correlation ratio between DR and macroeconomic factor is achieved with a lag of L1. When a two-factor model is constructed, the overall effectiveness of the model with lag L2 is greater than that of the model with lag L1 (although the variable correlation level with lag L2 with DR is slightly lower than the variable correlation with lag L1). In this case, choosing a variable with lag L2 it is necessary to ensure that the lag length is sufficiently interpretable. A deviation of 0.1 from the maximum correlation ratio of this factor with DR level is accepted as the interpretability measure.

[3] We used monthly points obtained by the linear interpolation method, and lying between quarterly observations as the training selection. In order to verify the quality of the model, the testing selection was built on the basis of quarterly observations.

Table 13. Values of indicator R^2 arranged by the years (%).

Indicator	2012	2013	2014	2015	2016	Total R^2
R^2	20.0	87.7	93.5	94.1	83.2	88.3

Table 14. Values of parameters of the Vasicek model (%).

ρ	DR_{avg}	$\bar{X}$	sigma
8.49	4.78	0.32	1.71

Table 15. Scenario values of change of the annual GDP growth rate and one-year DR (segments of Trade and Manufacturing and Services) (%).

Scenario of change of the annual GDP growth rate	Indicator	2018	2019	2020 et seq.
Basic (50%)	GDP growth rate	1.60	1.20	1.10
	DR_{New}	2.43	2.87	4.68
Optimistic (25%)	GDP growth rate	2.90	1.90	
	DR_{New}	1.38	2.14	4.68
Worst-case (25%)	GDP growth rate	0.90	–2.90	
	DR_{New}	3.24	12.14	4.68
Annual DR, weighed on the basis of scenarios probability	DR_{New}	2.37	5.01	4.68

Figure 12. Comparison of the actual dynamics of the default rate in the segments of Trade and Manufacturing and Services with the forecasted rate.

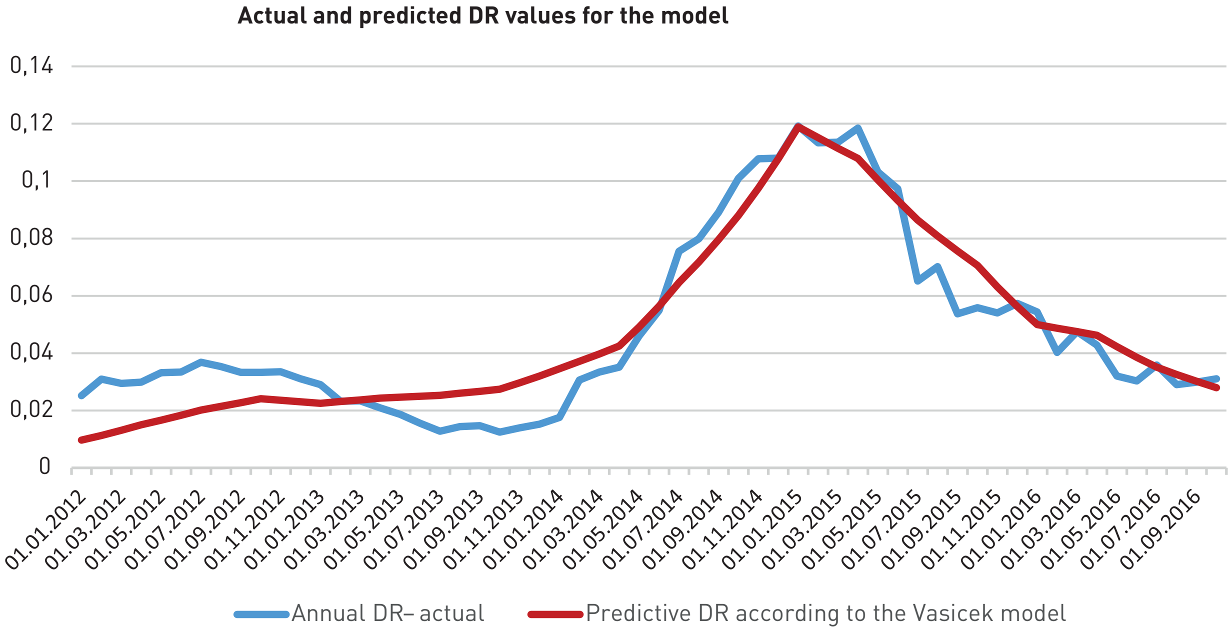


Table 14 presents information on the obtained values of parameters of the Vasicek model described in formula (17).

Graphically the comparison of the actual dynamics of the default rate in the segments of Trade, and Manufacturing and Services with the forecasting rate is presented as follows (Figure 12).

Choice of Scenarios of Parameters Change

In order to take into consideration the forecasting macroeconomic information for two years we took as the basic, optimistic and worst-case scenarios the forecasts of the annual GDP growth rate on the basis of statistics published by the Bank of Russia [12].

In accordance with the explanations given by the Impairment Transition Group (ITG), the ECL evaluation should take into account at least two macroeconomic scenarios if there is a significant non-linear interrelation between macro parameters in various possible scenarios and credit losses related to them. For the segments of Trade and Manufacturing and Services this interrelation is non-linear. It is expressed in the nonlinear nature of the type of DR dependence in the level of the annual GDP growth rate which, taking into consideration the actual data, shows a more intensive growth of DR level in an unfavourable economic environment. On the contrary, in a favourable economic environment DR decreases less intensively, approaching asymptotically 0% (Figure 13) [9].

The probability of alternative scenarios is chosen in accordance with an expert opinion of the bank taking into consideration the following rules:

- The probability of the optimistic worst-case scenarios should be less than the probability of the basic scenario;
- The sum of probabilities of all scenarios should be 100%.

In the general case the bank accepts the probability of optimistic and worst-case scenarios as equal. But asymmetrical scenarios are possible if, according to the bank's expert opinion one of the alternative scenarios of displacement against the basic scenario seems to be more probable: optimistic or worst-case scenario.

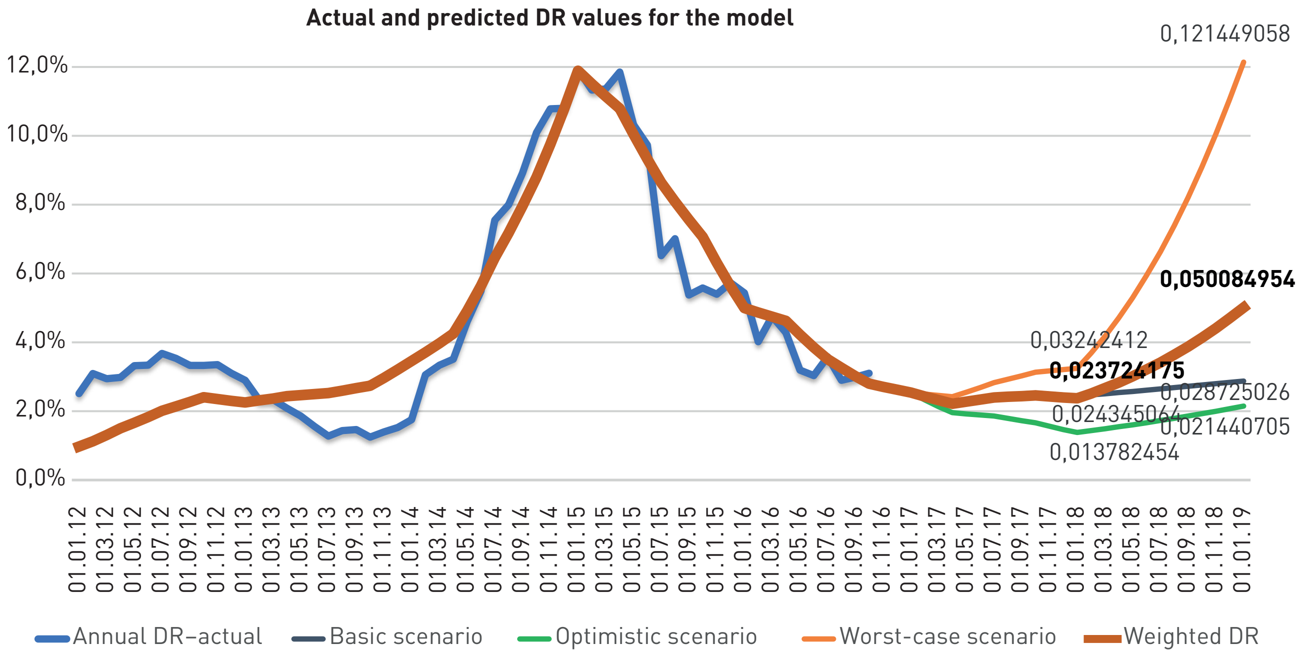
The probabilities of macroeconomic scenarios should be equal for all segments and macroeconomic indicators.

Calculations of the optimistic and worst-case forecasts of the GDP growth rate and corresponding forecast of DR for the segments of Trade and Manufacturing and Services built on the basis of the macroeconomic Vasicek model described in item 10.1.2 are represented in Table 15 and Figure 13.

The influence of macroeconomic factors is taken into consideration only for the first and second years. From the third year onwards, the value of the central tendency is used because PD PIT forecast for a period exceeding two years may be insufficiently reliable due to a decrease of the accuracy of macroeconomic forecasts when the forecasting horizon increases. In accordance with IFRS 9, an assessment of the expected credit losses does not require an obligatory detailed evaluation for periods reaching to the distant future. For such periods a company may extrapolate the existing reliable information [9].

For the purpose of defining Lt PD PIT, taking into consideration the predictive macroeconomic information, the average PD of the portfolio is accepted as 4.68% (average one-year DR in the segments of Trade and Manufacturing and Services, data from 01.11.2011 to 01.10.2016). The forecasting default rates weighed on the basis of scenarios probability for 2018 amounted to 2.37%, and for 2019 – 5.01%.

Figure 13. Graphical interpretation of macroeconomic scenarios



The offered macroeconomic model ensures a high stability of predicted values depending on the level of input from macroeconomic factors. This is provided by the choice of the type of the function of dependence of DR on GDP according to the Vasicek model which is close to the line function at medium and high DR but at the same time provides for a gradual asymptotic approximation of DR to zero when the GDP growth forecast is 3% and more. On the basis of testing results the predictive power of the model (presented as the determination coefficient R^2) is stable from year to year. This is confirmed by a small value of standard deviation of the determination coefficient R^2 which equals 5.2%.

Correction of TTC Lt PD for taking the macroforecast into consideration

The correction of the final TTC Lt PD (for the first and second year) presented in item 2.4 was effected in accordance with formula (15).

Table 16 represents the final one-year conditional PD which indicates the probability of default taking into consideration the influence of macroeconomic information.

Table 17 presents the final one-year marginal PD, which indicates the probability of default taking into consideration the influence of macroeconomic information and participation in ECL evaluation.

Table 16. The final one-year conditional PD, which indicates the probability of default taking into consideration the influence of macroeconomic information.

Scale	PD TTC	Forward PD (PD PIT for the 1st and 2nd year, PD TTC for the 3–5 years)				
		Year 1	Year 2	Year 3	Year 4	Year 5
1+	0.01	0.00	0.01	0.01	0.01	0.01
1	0.02	0.01	0.02	0.02	0.02	0.02
1–	0.04	0.02	0.04	0.04	0.04	0.04
2+	0.08	0.04	0.09	0.08	0.08	0.08
2	0.16	0.08	0.17	0.16	0.16	0.16
2–	0.32	0.16	0.34	0.32	0.32	0.32
3+	0.45	0.22	0.53	0.44	0.39	0.35
3	0.58	0.29	0.88	0.73	0.65	0.59
3–	0.75	0.37	1.46	1.22	1.09	0.98
4+	0.96	0.48	2.44	2.03	1.81	1.64
4	1.23	0.61	2.55	3.01	3.13	3.08
4–	1.58	0.79	5.08	4.73	4.37	3.99
5+	2.03	1.02	5.08	4.73	4.37	3.99
5	2.61	1.31	6.62	6.62	6.23	5.73
5–	3.36	1.69	6.81	6.81	6.43	5.92
6+	4.31	2.18	8.61	7.88	7.13	6.39
6	5.54	2.82	10.89	9.13	7.90	6.90
6–	7.12	3.66	11.76	9.94	8.60	7.49
7+	9.14	4.74	12.71	10.81	9.35	8.14
7	11.74	6.18	13.74	11.77	10.17	8.84
7–	15.08	8.08	18.41	14.11	12.67	11.50
8+	19.37	10.63	24.64	16.91	15.78	14.95
8	24.89	14.09	32.94	20.27	19.66	19.45
8–	31.97	18.87	43.95	24.30	24.48	25.30
9	41.06	25.64	58.48	29.13	30.49	32.91

Table 17. The final one-year marginal PD, which indicates the probability of default taking into consideration the influence of macroeconomic information (segment of Trade) (%).

Scale	PD TTC	MPD				
		Year 1	Year 2	Year 3	Year 4	Year 5
1+	0.01	0.00	0.01	0.01	0.01	0.01
1	0.02	0.01	0.02	0.02	0.02	0.02
1–	0.04	0.02	0.04	0.04	0.04	0.04
2+	0.08	0.04	0.09	0.08	0.08	0.08
2	0.16	0.08	0.17	0.16	0.16	0.16
2–	0.32	0.16	0.34	0.32	0.32	0.32
3+	0.45	0.22	0.52	0.43	0.39	0.35
3	0.58	0.29	0.88	0.72	0.64	0.57
3–	0.75	0.37	1.46	1.20	1.05	0.94
4+	0.96	0.48	2.43	1.97	1.73	1.53
4	1.23	0.61	2.54	2.92	2.94	2.80
4–	1.58	0.79	5.04	4.45	3.92	3.43
5+	2.03	1.02	5.03	4.46	3.93	3.44
5	2.61	1.31	6.53	6.10	5.37	4.62
5–	3.36	1.69	6.69	6.24	5.49	4.73
6+	4.31	2.18	8.42	7.05	5.87	4.89
6	5.54	2.82	10.58	7.91	6.22	5.00
6–	7.12	3.66	11.33	8.45	6.58	5.24
7+	9.14	4.74	12.11	8.99	6.94	5.47
7	11.74	6.18	12.89	9.53	7.27	5.67
7–	15.08	8.08	16.92	10.58	8.16	6.47
8+	19.37	10.63	22.02	11.39	8.83	7.05
8	24.89	14.09	28.30	11.69	9.06	7.23
8–	31.97	18.87	35.65	12.09	9.38	7.48
9	41.06	25.64	43.48	12.66	9.82	7.84

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Appendix 1. Bank Master Scale (%)

Scale of PJSC BANK XXX	Probability of default (PD)	Lower limit of the probability of default	Upper limit of the probability of default
1+	0.01	0.00	0.02
1	0.02	0.02	0.03
1–	0.04	0.03	0.07
2+	0.08	0.07	0.14
2	0.16	0.14	0.27
2–	0.32	0.27	0.38
3+	0.45	0.38	0.49
3	0.58	0.49	0.63
3–	0.75	0.63	0.81
4+	0.96	0.81	1.04
4	1.23	1.04	1.33
4–	1.58	1.33	1.71
5+	2.03	1.71	2.20
5	2.61	2.20	2.82
5–	3.36	2.82	3.63
6+	4.31	3.63	4.66
6	5.54	4.66	5.99
6–	7.12	5.99	7.69
7+	9.14	7.69	9.88
7	11.74	9.88	12.69
7–	15.08	12.69	16.30
8+	19.37	16.30	20.93
8	24.89	20.93	26.89
8–	31.97	26.89	34.54
9	41.06	34.54	100
10	100	100	100

Appendix 2. Glossary. Terms and Definitions

Probability of default	–	probability (in percent) of default occurrence as regards the customer's obligations within one year, defined by means of the model of the probability of default evaluation
Internal credit rating	–	indicator providing a comprehensive characteristic of the customer's/project's creditworthiness, calculated on the basis of the risk factor indicators
Default	–	failure to fulfill obligations of loan repayment by the borrower (default is taken into consideration in accordance with the definition stated in the article)
Cumulative probability of default (cPD)	–	probability of default at any moment within the period T (accumulated probability of default)
Marginal probability of default, mPD (t)	–	unconditional probability that default will occur within the future period t which is a part of the period T
Observation	–	data aggregate concerning a customer/project as of a certain date
Rating	–	In accordance with the Report on Development of the Inner Model of Evaluation of the Probability of Default of Corporate Borrowers in the Trade Segment
Rating group	–	an aggregate of several rating grades located in the rating scale at neighbouring positions unified in order to ensure a sufficient number of observations for a statistical analysis
Risk segment	–	a group of rating objects defined in accordance with inertial regulatory documents of the bank based on requirements of Basel Standards and Standards of the Bank of Russia
Conditional probability of default, PD (t)	–	the conditional probability that default will occur within the future period t which is a part of the period T, provided the default does not take place before the beginning of period t
Rating scale	–	Gradation rating scores in accordance with Appendix 1
Probability of default for the life-time of the instrument, Lt PD	–	Probability of default within the contractual validity term of a financial instrument

Designations and Abbreviations

cDR	–	cumulative default rate
cPD	–	cumulative probability of default
Dpd	–	days past due
DR	–	default rate
mPD	–	marginal probability of default
PD	–	probability of default
PD for 12 months	–	probability of default within 12 months after the reporting date
PIT (point-in-time)	–	calibration at the "point-in-time"
TTC	–	through-the-cycle
CPI	–	consumer price index
IFRS 9	–	International Financial Reporting Standards (IFRS) 9 Financial Instruments
ECL	–	expected credit losses
SP AACR	–	Software Package Accounting and Analysis of Credit Risks

APPENDIX 3. Calculation of influence of macroeconomic information based on integrated data in the segments of Trade, Manufacturing, and Services